Warsaw University of Technology

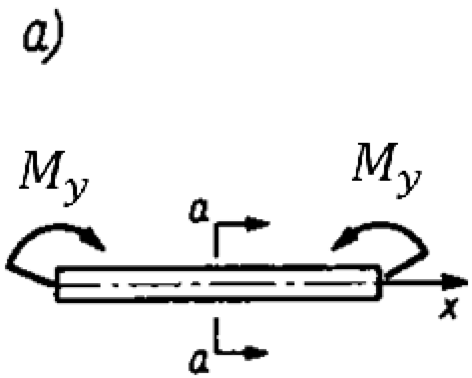
Faculty of Power and Aeronautical Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

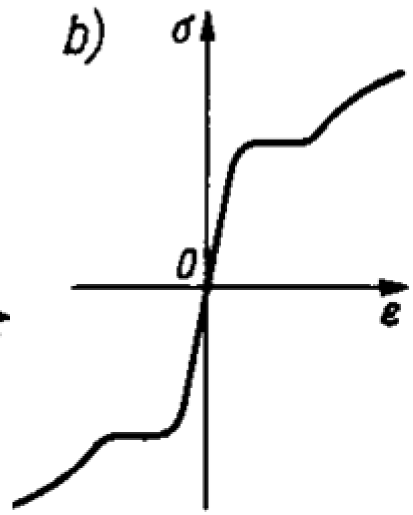
Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM 2)

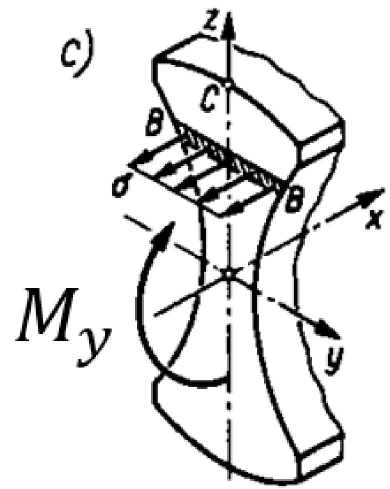
Elasto-plastic bending of a beam



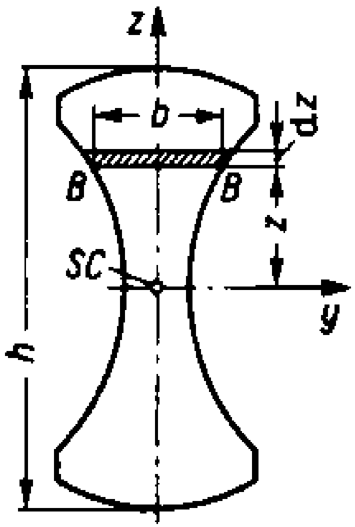
Pure bending



Tensile test graph



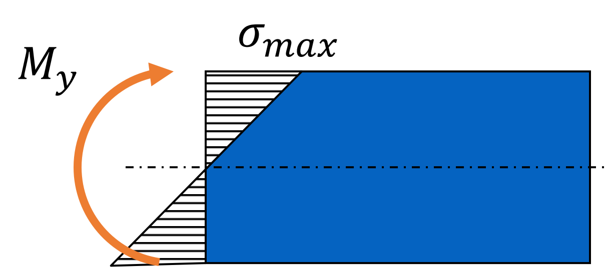
Normal stress in the cross-section



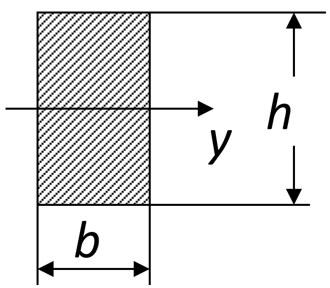
Cross-sectional area

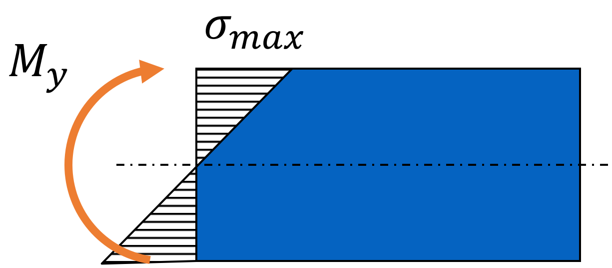
Stress distribution in the elastic range:

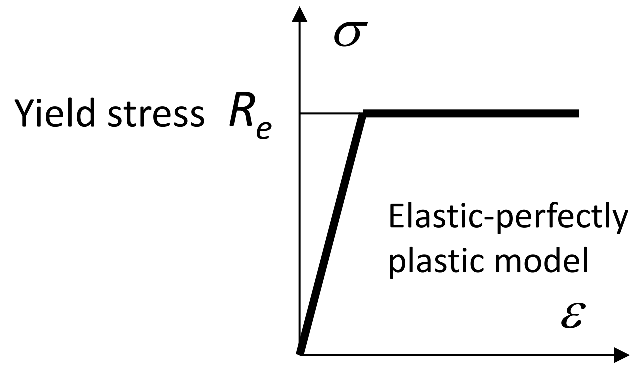
$$\boxed{\sigma = -\frac{M_y \cdot z}{J_y}} \rightarrow \boxed{\sigma_{max} = \frac{M_y \cdot \frac{h}{2}}{J_y}}$$

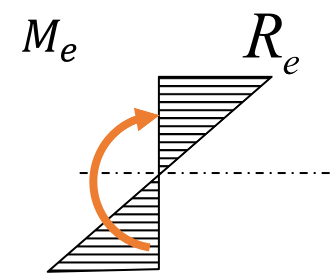


Bending of a beam with a rectangular cross-section made of a material with elasto-plastic characteristics without hardening:









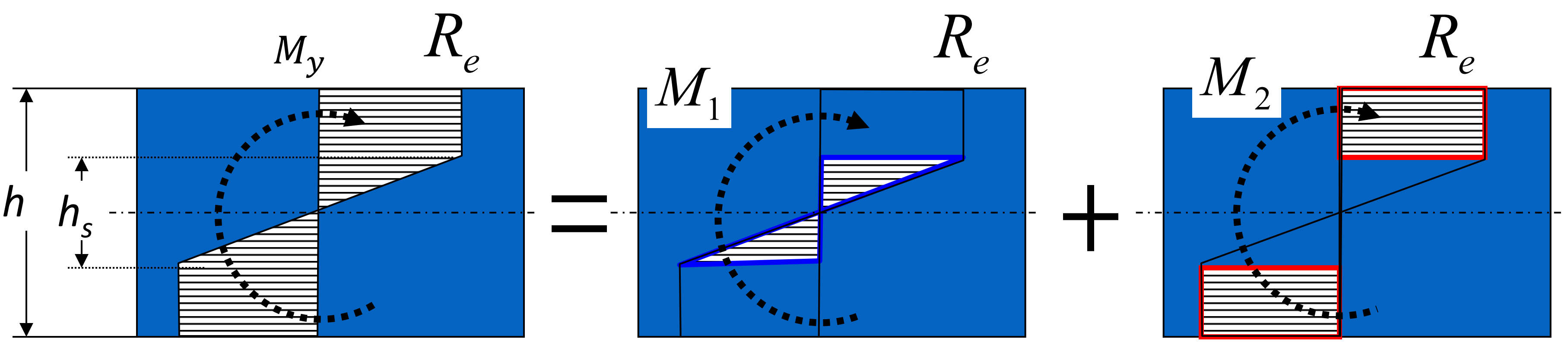
The maximum elastic moment M_e for a rectangular cross-section:

$$\sigma_{max} = \frac{M_y \cdot \frac{h}{2}}{\frac{bh^3}{12}} = R_e$$

$$\longrightarrow$$

$$\boxed{M_e = \frac{R_e b h^2}{6}}$$

After exceeding the maximum elastic moment, subsequent fibers will reach the plastic state and the stress distribution in the cross-section will be trapezoidal:

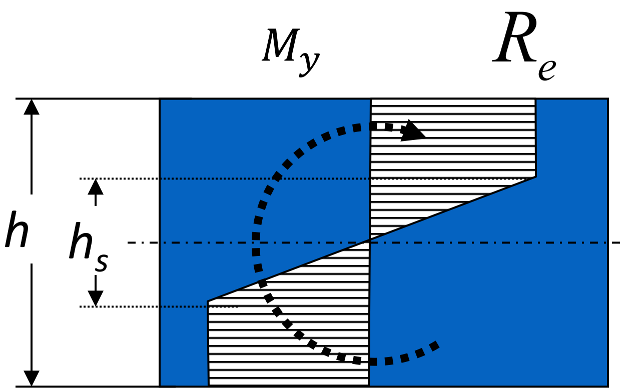


The cross-sectional area carry two parts of the bending moment.

After integration:

$$M_y = M_1 + M_2 = \boxed{\frac{R_e h_s^2 b}{6}} + \boxed{b \cdot R_e \frac{(h - h_s)}{2} \cdot \frac{(h + h_s)}{2}}$$

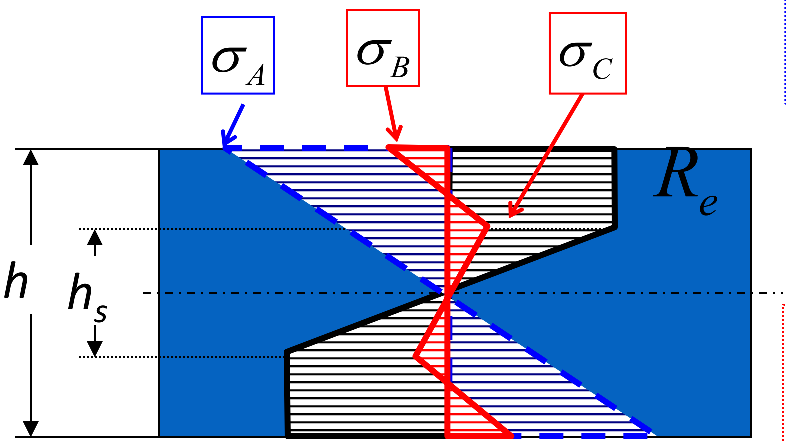
$$M_y = \frac{R_e b h^2}{6}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{h_s}{h}\right)^2\right] \rightarrow \boxed{M_y = M_e\left[\frac{3}{2} - \frac{1}{2}\left(\frac{h_s}{h}\right)^2\right]}$$



The bending moment for elasto-plastic bending:

$$\boxed{M_y = \frac{R_e b h^2}{6}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{h_s}{h}\right)^2\right]}$$

If we now unload the beam, this process will be elastic:



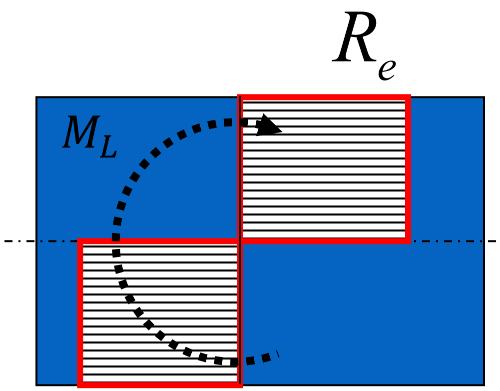
$$\sigma_A = \frac{6M_y}{bh^2} \rightarrow \boxed{\sigma_A = \tfrac{1}{2} R_e\left[3 - \left(\frac{h_s}{h}\right)^2\right]}$$

$$\sigma_B = \sigma_A - R_e \rightarrow \boxed{\sigma_B = \tfrac{1}{2} R_e\left[1 - \left(\frac{h_s}{h}\right)^2\right]}$$

$$\sigma_C = \frac{M_y \frac{h_s}{2}}{\frac{bh^3}{12}} - R_e \rightarrow \boxed{\sigma_C = \tfrac{1}{2} R_e\left[3\cdot\left(\frac{h_s}{h}\right) - \left(\frac{h_s}{h}\right)^3 - 2\right]}$$

After unloading, residual stresses remains

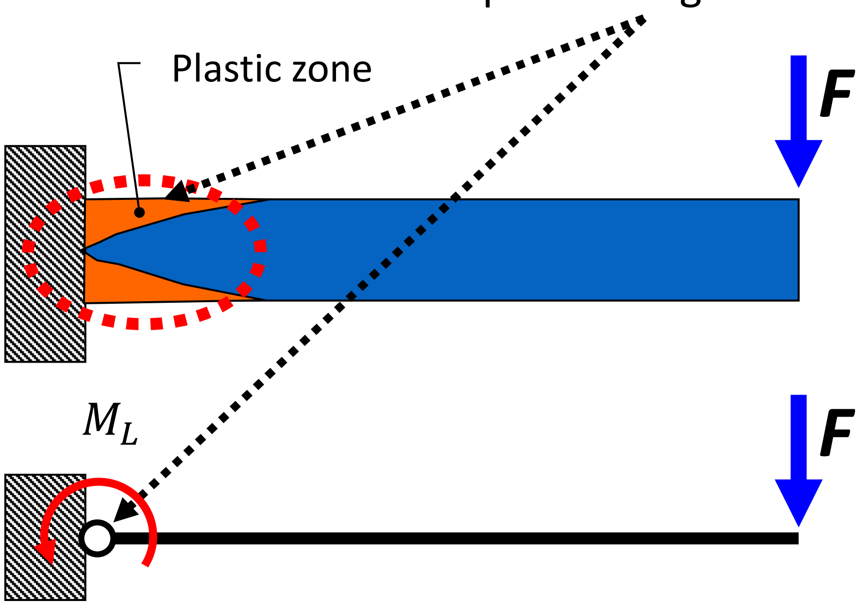
For full plasticization of the cross-section we need a limit moment M_L (corresponding to the beam's load-bearing capacity):



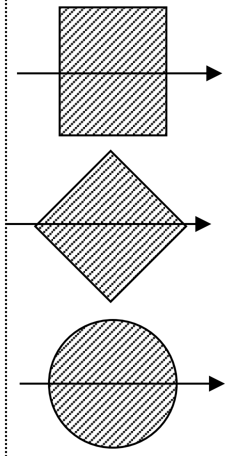
$$M_y = M_e\left[\frac{3}{2} - \frac{1}{2}\left(\frac{h_s}{h}\right)^2\right]$$

$$\boxed{M_L = M_y(h_s = 0) = 1.5M_e}$$

In the case of a cantilever beam, complete plasticization of the cross-section will result in the formation of a plastic hinge and the beam will become a mechanism.



For typical sections:



$$\boxed{M_L = 1.5M_e}$$

$$\boxed{M_L = 2.0M_e}$$

$$\boxed{M_L = 1.7M_e}$$